GAINS TO PRODUCERS FROM THE CARTELIZATION OF EXHAUSTIBLE RESOURCES

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I. Introduction

IT has been suggested that one reason for OPEC's formation and later success in maintaining itself as a cohesive cartel is that the gains from cartelization in the world oil market are so large. Indeed, looking at the historical success record of various cartels (or attempts at cartelization), a case could be made that those cartels were most successful for which potential monopoly profits were the greatest.[1] After all, there are significant costs associated with cartelization—political costs, cost of coordination of output and price, and, for each producer, costs associated with the risk of being undercut and losing significant short-term profits. Bearing these costs would not be worthwhile if the expected resulting gains were not at least as large. It is therefore important to know what the potential gains from cartelization are.

Often the analysis of producer gains from cartelization is based on a simple static computation of the potential monopoly profit in a particular market, given elasticity estimates for demand and for supply from those producers who would not be likely to join the cartel. Such a static analysis might be realistic and quite sufficient for such markets as bananas, coffee, and sugar, where demand and supply can adjust quickly to price, and where resource exhaustion is not a determinant of, nor constraint on, production. A static analysis might be misleading, however, for such markets as petroleum, copper, or bauxite. First, the process of reserve depletion for these resources might have an important impact on monopoly pricing decisions and on the potential gains from cartelization. Second, the markets for these resources are characterized by demands and supplies that adjust only slowly to changes in price, so that a cartel might have the potential for large short-term monopoly profits by taking advantage of adjustment lags.

The problem of measuring the potential gain from the cartelization of a particular exhaustible resource can be put quite simply. Given reserve levels and the dynamic structure of demand and cost, and given that the objective of each producer is to maximize the sum over time of discounted profits, what are the optimal price trajectories under competition and under cartelization, and how much larger is the sum of discounted profits as a result of cartelization? We assume that the (potential) cartel in question is able to behave as a perfect monopolist that knows the structure of demand and cost. In addition, we do not address the issue of how the gains from cartelization are to be divided up among the cartel members.

To measure these potential gains we turn to the theory of exhaustible resource pricing. It was Hotelling (1931) who first demonstrated that if extraction costs are constant, under competition, price minus marginal cost should rise at the rate of discount, while in a monopolistic market, rents (marginal revenue minus marginal cost) should rise at the rate of discount.[2] It can be shown that under conditions of constant demand elasticity and zero extraction costs, the price trajectories will be the same in both the monopoly and

Received for publication August 13, 1976. Revision accepted for publication May 4, 1977.

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This work was supported by the National Science Foundation under Grant No. SIA75-00739, and is part of a larger project to develop analytical models of the world oil market. The author would like to express his appreciation to Esteban Hnyilicza, who developed the computer program used for this work and provided assistance in obtaining computational solutions, and to Steve Martin, who pointed out an error in an earlier draft of this paper. Considerable thanks are also due to Ross Heide and Kai Wong for their excellent research assistance, and to M. A. Adelman, Michael Athans, James Burrows, Partha Dasgupta, Dermot Gately, Richard Gilbert, Henry Jacoby, Dale Jorgenson, David Kendrick, Edwin Kuh, Alan Manne, Robert Solow, and Joseph Stiglitz for their comments, criticisms, and suggestions.

[1] A historical summary of the experience of some international cartels is given in Eckbo (1975).

[2] For another derivation and interpretation of Hotelling's results, see Herfindahl (1967) and Gordon (1967). For further discussion, see Solow (1974) and Banks (1974).

competitive cases.[3] However, if extraction costs are positive and/or the elasticity of demand is rising, the monopoly price will initially be higher (and later be lower) than the competitive price, i.e., the monopolist will be relatively "conservationist."[4]

The relevant question, however, is *to what degree* will the price trajectories in the two cases differ? Stiglitz (1976) claims that in general "there is a very limited scope for the monopolist to exercise his monopoly power." This may be the case for some exhaustible resources, but not for others. It will depend in part on the particular way in which demand elasticities change and production costs increase as the resource reserve base is depleted. It will also depend on the ability of the monopolist to take advantage of adjustment lags in the demand for his output.

This paper attempts to calculate optimal monopolistic and competitive price trajectories for several exhaustible resource cartels, and thereby to determine the potential gains to producers from cartelization. Our model of cartelization is rather simple; we treat the cartel as a pure monopolist holding a known quantity of reserves and facing a "net demand" function (total world demand minus supply from "competitive fringe" producers who are not members of the cartel). We ignore such potential problems as differences in production costs for different members, differences in objectives, need for output rationalization, etc.[5] The approximate analysis is still useful, however, since if the gains to the pure monopolist are not large, we would not expect the more "realistic" cartel to remain stable over a long period of time, while if the gains are quite large, there should be sufficient incentive for the producers to overcome the problems typical of cartelization.

We limit ourselves here to three cartels that have already identified themselves as politically (if not economically) feasible realities. In particular, we consider OPEC in the case of petroleum, CIPEC (International Council of Copper Exporting Countries) in the case of copper, and IBA (International Bauxite Association) in the case of bauxite. It is clear that OPEC has already demonstrated its ability to enjoy large monopoly profits, and here we calculate the potential profits that it might enjoy in the future. The argument has been made, however, that OPEC is an exception to the rule, and that CIPEC, IBA, and other real or imagined natural resource cartels do not have the potential for any significant monopoly profits.[6] Here we explore this argument quantitatively.

In the next section we present the basic optimal pricing model for an exhaustible resource cartel facing a dynamic "net demand" function, and explain how optimal price trajectories for both the monopoly and competitive cases can be obtained. Then, the basic model is applied to OPEC and the world oil market, and is used to calculate optimal price trajectories for OPEC and measure OPEC's potential monopoly gains. Next we turn to the bauxite and copper markets, fitting the basic model to each in an attempt to measure the potential gains that IBA and CIPEC might realize by following optimal pricing rules. We conclude with some remarks about actual and potential cartel behavior.

II. An Optimal Pricing Model for an Exhaustible Resource

The basic model is specified to account for differences between short-run and long-run price elasticities both in demand and supply from "competitive fringe" countries. Total demand (TD) for the resource in question would be of the form

$$TD_t = f_1(P_t, Y_t, TD_{t-1}) \tag{1}$$

where P_t is real price and Y_t is the measure of aggregate income or product. This specification of the demand function takes into account the substitution of other materials for this resource (e.g., coal for oil or aluminum for copper), and since we assume that the prices of competing materials are fixed, they need not be included explicitly in (1).

[3] A nice demonstration of this is given by Stiglitz (1976).

[4] This is examined by Stiglitz (1976) and Sweeney (1975).

[5] The effects of these problems on pricing and output policy for OPEC are examined in Hnyilicza and Pindyck (1976).

[6] See, for example, Krasner (1974) and Smart (1975). For opposing views see Bergsten (1973, 1974) and Mikdashi (1974).

Net demand facing the cartel is

$$D_t = TD_t - S_t \tag{2}$$

where S_t is the supply function for the "competitive fringe," and is given by

$$S_t = f_2(P_t, S_{t-1}). \tag{3}$$

Resource depletion might be as significant a factor for the competitive fringe as it is for the cartel, in which case we can modify the supply function so that it moves to the left (rising marginal and average cost) in response to cumulative production CS:

$$S_t = f_2(P_t, S_{t-1}) \cdot (1+\alpha)^{-CS_t/\bar{S}} \tag{3'}$$

with

$$CS_t = CS_{t-1} + S_t, \tag{4}$$

and $\bar{S}$ is average annual competitive production and α is a parameter that determines the rate of depletion.

An accounting identity is needed to keep track of cartel reserves, R:

$$R_t = R_{t-1} - D_t. \tag{5}$$

The objective of the cartel is to pick a price trajectory $\{P_t\}$ that will maximize the sum of discounted profits:

$$\max W = \sum_{t=1}^{N} (1/(1+\delta)^t)[P_t - m/R_t]D_t \tag{6}$$

where m/R_t is average production cost (so that the parameter m determines initial average cost), δ is the discount rate, and N is chosen to be large enough (40–60 years) to approximate the infinite-horizon problem.[7] Since average costs become infinite as the reserve base R_t approaches zero, the resource exhaustion constraint need not be introduced explicitly.[8] As a result, we have a classical, unconstrained discrete-time control problem for which numerical solutions can be obtained easily.[9]

The solution to the above problem yields an optimal price trajectory $\{P_t^*\}$ and optimal sum of discounted profits W^* for the monopolist. We would like to compare these with the optimal price trajectory and sum of discounted profits that would result if the cartel dissolved (or never formed), and its member producers behaved competitively. We say "optimal" because competitive producers must still manage an exhaustible resource, balancing profits this year against profits in future years.

Although competitive producers cannot collectively set price, they each determine output given a price. We show in the appendix that the rate of output should be such that the competitive price satisfies the difference equation

$$P_t = (1+\delta)P_{t-1} - \delta m/R_{t-1}. \tag{7}$$

If this were not the case, larger profits could be obtained by shifting output from one period to another. In addition, the initial price must be such that two constraints hold. First, the resulting price and output trajectories $\{P_t\}$ and $\{D_t\}$ must both satisfy, at every point in time, net demand as given by equations (1), (2), and (3), i.e., supply and demand must be in market equilibrium. Second, as the price rises monotonically over time, profits must become zero at the same time that net demand goes to zero.[10] If demand becomes zero before profits do,

[7] Note that we are treating the fringe and the cartel asymmetrically. We might expect members of the fringe to set output optimally, as does the cartel, with the same price foresight that the cartel has. Including this consideration, however, would considerably complicate the computational solution.

[8] A constraint on the production capacity of the cartel can be introduced implicitly by rewriting the objective function as

$$\max W = \sum_{t=1}^{N} (1/(1+\delta)^t)[P_t - m/R_t - a\exp(bD_t^c)] \tag{6'}$$

where c is large enough so that the added expression is close to zero when D_t is just below the capacity constraint and very large when D_t is just above. Typically, c equal to 20 or 30 (with a and b chosen appropriately) is sufficient. In the applications of this model that follow, however, the capacity constraint is never even approached (i.e., it is always optimal for the cartel to produce at below capacity), so that (6′) need not be used.

[9] We use a general nonlinear optimal control algorithm developed by Hnyilicza (1975). Using that algorithm, optimal pricing policies can be derived in a classical control theoretic framework for any model in implicit state-variable form:

$$g(x_t, x_{t-1}, P_{t-1}, z_{t-1}) = 0$$

where g is a vector of nonlinear functions in the set of state variables x_t (endogenous and lagged endogenous variables, together with state variables defined for P and elements of z occurring with lags longer than one period), the price (control) variable P_t, and a set of exogenous variables z_t. The objective function can also be general in form.

[10] We are assuming here that the net demand curve indeed intersects the vertical axis. If this were not the case, price would rise indefinitely with demand asymptotically approaching zero and exhaustion occurring at $t=\infty$.

some of the resource is wasted and yields no profits; profits would be greater if the resource were depleted more rapidly (at a lower price). If profits become zero before demand becomes zero, depletion is occurring too rapidly and should proceed more slowly.[11]

The computation of the optimal price trajectory for the competitive case is thus straightfoward. Pick an initial P_0 and *simulate* (i.e., solve over time) equation (7) together with equations (1), (2), (3), and (5). Repeat this for different values of P_0 until D_t and R_t become zero simultaneously.

III. Petroleum—The Gains to OPEC

The basic model described above is parameterized for the world oil market so that it is consistent with the reserve, production, and elasticity estimates of OECD (1974), and with crude elasticity estimates obtained from aggregate time series data.[12] The equations of the model are as follows:

$$TD_t = 1.0 - .13P_t + .87TD_{t-1} + 2.3(1.015)^t \quad (8)$$

$$S_t = (1.1 + .10P_t)\cdot(1.02)^{-CS_t/7} + .75S_{t-1} \quad (9)$$

$$CS_t = CS_{t-1} + S_t \quad (10)$$

$$D_t = TD_t - S_t \quad (11)$$

$$R_t = R_{t-1} - D_t \quad (12)$$

$$\max W = \sum_{t=1}^{N} ((1+\delta)^t)^{-1}[P_t - 250/R_t]D_t \quad (13)$$

where

TD_t = total demand for oil (billions of barrels (bb) per year)
D_t = demand for cartel oil (bb/yr.)
S_t = supply of competitive fringe (bb/yr.)
CS_t = cumulative supply of competitive fringe (bb)
R_t = reserves of cartel (bb)
P_t = price of oil (\$ per barrel) in constant 1975 dollars.

The linear demand equation (8) is based on a total demand of 18 billion barrels per year at a price of \$6 per barrel; at that price the short-run price elasticity is 0.04 and the long-run elasticity is 0.33 (with a Koyck adjustment), while at a \$12 price the short-run and long-run elasticities are 0.09 and 0.90, respectively. The last term in equation (8) provides an autonomous rate of growth in demand of 1.5% per year, corresponding to a long-run income elasticity of 0.5 and a 3% real rate of growth in income. (The use of an isoelastic instead of linear demand function allows for somewhat higher cartel prices, but as we find empirically, the difference is small.)

At a \$6 price competitive supply is about 6.5 billion barrels per year. The supply equation (9) implies short-run and long-run elasticities of 0.09 and 0.35, respectively, at the \$6 price, and 0.16 and 0.52, respectively, at a \$12 price. Depletion of competitive fringe reserves pushes the supply function to the left over time. After a cumulative production of 210 billion barrels (e.g., 7 bb/yr. for 30 years) supply would fall (assuming a fixed price) to 55% of its original value.[13] We assume that no "backstop technology" (e.g., shale oil) would add significantly to competitive supply at prices below \$20.

Note that the average cost of production for the OPEC cartel rises hyperbolically as R_t goes to zero. The initial reserve level is taken to be 500 billion barrels, and initially average cost is 50¢ per barrel.[14]

In calculating optimal price policies we take as initial conditions $TD_0 = 18.0$, $S_0 = 6.5$, and $CS_0 = 0$. Price trajectories are calculated over a forty-year horizon, with discount rates of 0.05

[11] Note that if the autonomous rate of growth in net demand is greater than the discount rate δ, the optimal price will be such that output is always infinitesimally small, since then it always pays to postpone *all* production into the future.

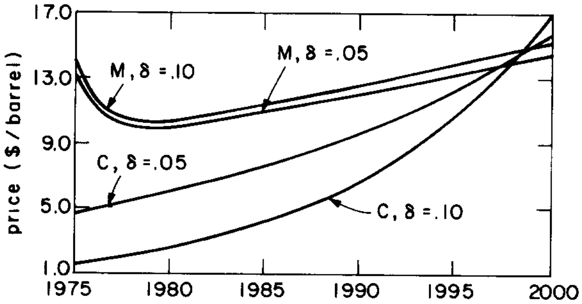
[12] Cremer and Weitzman (1976) and Kalymon (1975) have recently constructed similar optimal pricing models for OPEC. Those studies, however, do not account for adjustment lags. Non-optimizing simulation models of OPEC behavior have also been constructed; see, for example, Blitzer et al. (1975), and Fischer et al. (1975).

[13] Note that there is no fixed upper bound on cumulative production by competitive fringe countries; there is always *some* price at which additional supplies would be forthcoming. For example, after 210 billion barrels have been produced, a price of \$18.5 would be needed to maintain production at 6.5 bb/yr., while after 420 billion barrels have been produced a \$43 price would be needed to maintain the 6.5 bb/yr. production level.

[14] 500 billion barrels represents a rough estimate of *proved* reserves for OPEC (source: *Oil and Gas Journal*, Dec. 1975). Potential reserves would be greater, but we assume that the more conservative number would be used by OPEC to plan a long-term price policy.

and 0.10.[15] Optimal prices for both the monopolistic and competitive cases are shown graphically for the first 25 years in figure 1, and prices, total demand, OPEC production, OPEC reserves and discounted profits are shown for the longer horizon in table 1.

FIGURE 1.—OPTIMAL OIL PRICE TRAJECTORIES
(M = monopoly, C = competition)



Observe that the optimal monopoly price is $13 to $14 in the first year (1975), declines over the next five years to around $10, and then rises slowly. This price pattern is a characteristic result of incorporating adjustment lags in OPEC's net demand function. If total demand and competitive supply had been modelled as static functions of price, the optimal monopoly price, like the competitive price, would rise monotonically. With adjustment lags, however, it is optimal for OPEC to charge a higher price initially, taking advantage of the fact that net demand can adjust only slowly. Of course, in the competitive case price will still rise monotonically, even with adjustment lags, since the competitors cannot together restrict output to take advantage of the initially inelastic demand.

Observe also that with the smaller discount rate the initial competitive price is higher. With a large discount rate producers pay less attention to the future, so that their output decisions more closely approximate those that would be made without accounting for resource depletion, i.e., they more closely approximate the static competitive solution. When the discount rate is low producers must be more "conservationist," so that initial output levels

TABLE 1.—PETROLEUM

	P	TD	D	R	Π_d
		Monopoly: $\delta = .05$			
1975	13.24	17.24	9.94	488.5	126.5
1976	11.19	16.88	9.23	478.6	93.8
1977	10.26	16.72	8.94	469.3	78.9
1978	9.90	16.66	8.87	460.4	71.7
1979	9.82	16.66	8.91	451.5	67.9
1980	9.88	16.69	9.00	442.6	65.7
1985	10.84	16.96	9.67	396.3	60.6
1990	11.98	17.32	10.40	346.3	56.3
1995	13.18	17.74	11.15	293.0	51.8
2000	14.46	18.22	11.91	235.7	47.1
2005	15.92	18.75	12.66	174.6	42.5
2010	20.29	18.67	12.55	110.5	41.0
		Monopoly: $\delta = .10$			
1975	14.08	17.13	9.75	488.5	132.3
1976	11.75	16.71	8.94	478.8	91.2
1977	10.70	16.52	8.61	469.8	72.4
1978	10.28	16.44	8.52	461.2	62.3
1979	10.19	16.42	8.54	452.7	56.2
1980	10.26	16.43	8.61	444.1	51.8
1985	11.28	16.62	9.21	399.9	37.8
1990	12.51	16.90	9.87	352.6	27.9
1995	13.80	17.24	10.53	301.9	20.3
2000	15.18	17.63	11.20	248.0	14.6
2005	16.72	18.06	11.86	190.7	10.5
2010	20.52	18.05	11.88	130.5	7.9
		Competitive: $\delta = .05$			
1975	4.62	18.36	11.92	488.5	46.64
1976	4.85	18.68	12.29	476.6	48.22
1977	5.09	18.96	12.62	464.3	49.62
1978	5.35	19.20	12.90	451.7	50.90
1979	5.62	19.42	13.14	438.8	52.00
1980	5.90	19.60	13.35	425.6	52.92
1985	7.53	20.19	13.99	357.3	55.87
1990	9.60	20.30	14.04	287.1	56.15
1995	12.26	19.96	13.55	217.6	54.04
2000	15.65	19.13	12.48	151.8	49.15
2005	19.97	17.69	10.71	92.6	40.76
2010	25.48	15.46	8.07	44.0	27.59
		(depletion occurs in 2019)			
		Competitive: $\delta = .10$			
1975	1.55	18.76	12.63	488.5	11.92
1976	1.71	19.43	13.59	475.9	13.31
1977	1.88	20.03	14.40	462.3	14.49
1978	2.06	20.56	15.10	447.9	15.49
1979	2.27	21.04	15.70	432.8	16.50
1980	2.50	21.46	16.20	417.1	17.38
1985	4.02	22.87	17.74	332.4	20.32
1990	6.47	23.49	17.93	242.8	21.23
1995	10.43	22.64	16.74	154.9	19.94
2000	16.79	20.49	13.65	76.5	15.49
2005	27.05	15.98	7.72	18.7	5.50
2010					
		(depletion occurs in 2008)			

[15] With a discount rate of 0.02 or less the initial *competitive* price is over $10. With autonomous growth in net demand (resulting from autonomous growth in total demand and depletion of competitive fringe reserves) the optimal production rate will approach zero if the discount rate is small enough. Pre-OPEC oil prices were never in the vicinity of $10, which means that competitive producers either used higher discount rates or else used a low discount rate but produced at a very sub-optimal rate. The presence of risk (regarding potential reserves, changes in market conditions, etc.) would make it most reasonable to use a discount rate of 0.05 or higher, and so we use 0.05 and 0.10.

are reduced and prices are closer to those that would be set by a monopolist. As can be seen in the results, resource exhaustion by competitive producers occurs earlier at the higher discount rate, since prices begin low and then rise more rapidly in later years when the discount rate is large. The competitive solution —as given by equation (7)—is such that a lower discount rate, implying a lower rate of increase in price, will restrict initial output so that initial price is higher and closer to the monopoly price. With a 10% discount rate the competitive price is \$1.55 in the first year, which is in the vicinity of actual pre-1974 Persian Gulf prices. We might thus assume that prior to 1974, OPEC producers set output levels competitively using a high discount rate.

The relative gains from cartelization are summarized in table 2. Note that these gains are largest during the first five years, particularly when the discount rate is large, since it is during this period that a monopoly can take advantage of adjustment lags and reap large short-term profits. Even over the longer term, however, the sum of discounted profits is 50% to 100% larger under cartelization. The incentive for maintaining the cartel is thus considerable.

TABLE 2.—PETROLEUM: RELATIVE PROFITS (ratio of NPV^a for monopolist (M) to NPV for competitors (C))

	$NPV=\sum\Pi_d$ (1975 until exhaustion or 2015)
$\delta=.05$	$NPV_M/NPV_C=2092/1362=1.54$
$\delta=.10$	$NPV_M/NPV_C=1078/556=1.94$
	$NPV=\sum\Pi_d$ (first five years)
$\delta=.05$	$NPV_M/NPV_C=438.8/247.4=1.77$
$\delta=.10$	$NPV_M/NPV_C=414.4/71.7=5.78$

[a] NPV = net present value of profits.

Obviously these results are dependent on the particular model and parameter values described earlier. However, changing the model's parameters has only a small effect on the numerical results. For example, if the elasticities (short- and long-term) of total demand are *doubled*, optimal monopoly and competitive prices decrease by less than 20%. Doubling the elasticity of competitive supply results in a decrease in price of about 10%. Replacing the total demand and competitive supply equations with isoelastic equations (using the \$6 elasticities from the linear equations) or doubling the length of the lag response in total demand results in price trajectories that are at all times within 15% of those reported in table 1.[16] Doubling or halving OPEC's initial average production cost has less than a 5% effect on monopoly price and less than a 20% effect on competitive price. None of the aforementioned changes affects the ratios of sums of discounted profits reported in table 2 by more than 10%. The gains to OPEC from cartelization are thus high under a broad range of assumptions.

Changing the initial level of OPEC reserves from 500 billion barrels to 800 billion barrels (but increasing M, so that initial average cost is still 50¢) has almost no effect (less than 10%) on the optimal monopolistic price trajectories, but it does have a significant effect on the competitive solutions. Using the higher reserve estimate, initial competitive prices for $\delta=0.05$ and 0.10 are \$3.06 and \$0.65, respectively. These prices are lower because there is less need to conserve (exhaustion occurs after 43 years at the high discount rate and 54 years at the low rate, as compared to 33 years and 44 years in table 1), so that prices are closer to those that would result from competitors making static output decisions (i.e., setting output one period at a time, ignoring future periods). Since competitive prices and discounted profits are lower, the use of a higher initial reserve estimate means *greater potential gains from cartelization* (NPV_M/NPV_C is 2.42 for $\delta=0.05$ and 3.05 for $\delta=0.10$). The 800 billion barrel figure represents proven reserves plus "highly likely" potential reserves, and thus might be a more realistic number to use in setting price or output over time. We chose the 500 billion barrel figure in part because it better dramatizes the effects of depletion on price and output dynamics.

IV. Bauxite—The Gains to IBA[17]

There are no published econometric studies of the world bauxite market that can be tapped

[16] Note that net demand for OPEC oil will *not* be isoelastic.

[17] IBA countries in 1974 included Australia, Jamaica, Surinam, Guyana, Guinea, Yugoslavia, and Sierra Leone, and accounted for 74% of non-Communist world bauxite production.

for elasticity estimates, and therefore our competitive supply equation must be somewhat speculative. A total demand equation, however, can be derived from the demand for aluminum and from cost data on the production of alumina from bauxite and from alternative aluminum-bearing ores. Data on costs, reserves, and production are available from the U.S. Bureau of Mines (1974b, 1975). The equations of the model are listed below:

$$TD_t = \left[1.048 - .131P_t + 13.1(1.03)^t\right] \times e^{-(.0641P_t)^{10}} + .80TD_{t-1} \qquad (14)$$

$$S_t = (-1.69 + .4225P_t)\cdot(1.005)^{-CS_t/17} + .90S_{t-1} \qquad (15)$$

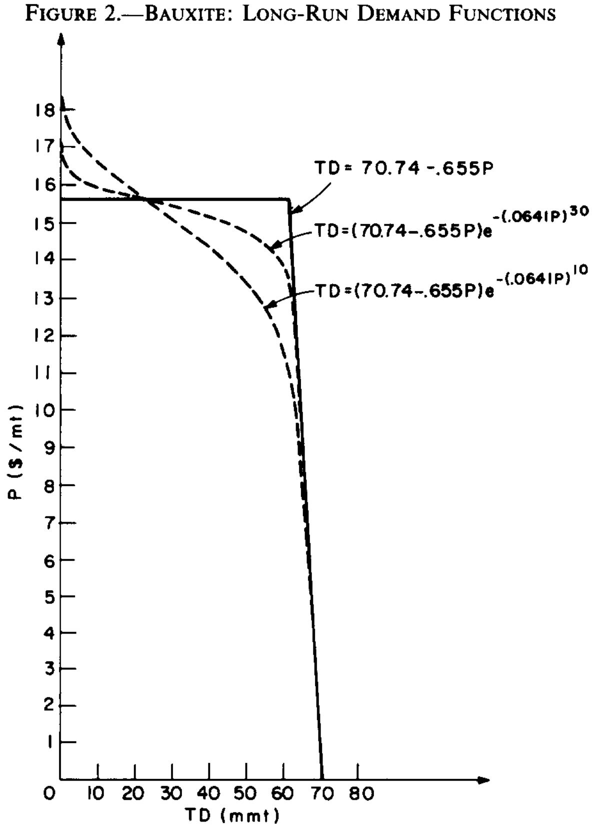
$$CS_t = CS_{t-1} + S_t \qquad (16)$$

$$D_t = TD_t - S_t \qquad (17)$$

$$R_t = R_{t-1} - D_t \qquad (18)$$

$$\max W = \sum_{t=1}^{N} ((1+\delta)^t)^{-1} \times [P_t - (55{,}000/R_t)]D_t \qquad (19)$$

FIGURE 2.—BAUXITE: LONG-RUN DEMAND FUNCTIONS



where

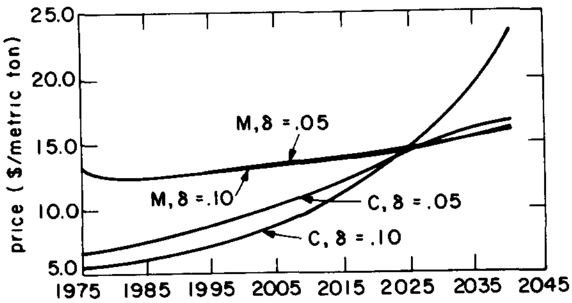
TD_t = total demand for bauxite (millions of metric tons per year (mmt/yr.))
S_t = supply from competitive fringe (mmt/yr.)
CS_t = cumulative supply of competitive fringe (mmt)
D_t = net demand for IBA bauxite (mmt/yr.)
R_t = reserves of cartel (mmt)
P_t = price of bauxite ($/mt) in *constant 1973 dollars*.

For a range of prices up to about $15.60 the demand for bauxite is quite inelastic, but at higher prices it becomes economical to produce alumina from sources other than bauxite, so that the demand for bauxite becomes almost infinitely elastic.[18] In the inelastic region the demand for bauxite depends on the demand for aluminum. At a bauxite price of $8 per ton, bauxite itself represents about 8% of the cost of producing aluminum. Using short- and long-run price elasticities of aluminum demand of −0.2 and −1.0, respectively, the corresponding price elasticities for bauxite would be −0.016 and −0.08. The income elasticities should be the same as those for aluminum; we use 0.2 and 1.0 for the short-run and long-run, respectively. The term $(1.03)^t$ in equation (14) builds in the assumption of 3% annual real growth in income.

At a price of around $15.60 we would expect the demand for bauxite to fall rapidly to zero.

[18] Other sources of alumina (Al_2O_3) include high-alumina clays, dawsonite, alumite, and anorthosite, all of which are in great abundance in the earth's crust. The most economical alternative to bauxite is to produce alumina from high-alumina clays using the hydrochloric acid–ion exchange process. In this process there is an operating cost of $74.5 per ton, of which $5.02 is the cost of clay at $1 per ton. In addition, the fixed capital cost of a 1,000 ton per day plant is $108 million. Assuming a 10% cost of capital and 350 operating days per year, and ignoring replacement costs, and/or maintenance, capital cost becomes $30.85 per ton of alumina, so that the total cost of producing alumina through this process is $105 per ton. Producing alumina from bauxite using the Bayer process would not be this expensive unless the price of bauxite rose to $15.60 per ton. At that price operating costs would be $86 per ton, of which $40 would be the cost of bauxite (about 2.5 tons of bauxite for every ton of alumina). The cost of a 1,000 ton per day plant is $66 million, so that the capital cost is $19 per ton, for a total cost of $105 per ton. (*All prices in 1973 dollars.*) Source of data: U.S. Bureau of Mines (1974a).

FIGURE 3.—OPTIMAL BAUXITE PRICE TRAJECTORIES
(M = monopoly, C = competition)



For this reason the exponential term is included in equation (14). By choosing a large enough exponent for $0.0641P_t = P_t/15.60$, we can achieve an arbitrarily close approximation to a piecewise linear demand function. In fact, we would expect demand to start falling off at prices somewhere below \$15.60 (if for no other reason than anticipation of future price increase), and demand to be small but not zero at higher prices, so we choose 10 as the exponent. The long-run demand function is plotted in figure 2.

The supply function for the competitive fringe is straightforward and is given by equation (15). At a price of \$8 the short-run and long-run elasticities are 0.2 and 2.0, respectively. Since there is considerable uncertainty as to what the true *long-run* elasticity

TABLE 3.—BAUXITE

	P	TD	D	R	Π_d[a]
		Monopoly: $\delta = .05$			
1975	13.03	62.9	43.9	10,956	352.1
1976	12.79	61.6	40.8	10,915	302.9
1977	12.62	61.0	38.8	10,876	267.0
1978	12.49	61.2	37.7	10,839	240.9
1979	12.40	61.7	37.2	10,801	223.4
1980	12.34	62.7	37.2	10,764	210.4
1985	12.37	70.2	41.7	10,567	183.5
1990	12.56	80.0	50.1	10,334	174.3
1995	12.79	91.1	60.5	10,053	167.0
2000	13.03	103.3	72.6	9,715	158.1
2005	13.27	116.6	86.1	9,312	146.6
2010	13.53	130.6	100.4	8,839	133.0
2015	13.82	144.8	114.9	8,293	117.3
2020	14.14	157.9	128.4	7,677	98.6
2025	14.50	168.0	138.8	7,002	80.4
2030	14.92	172.1	143.2	6,291	60.4
2035	15.38	167.4	138.6	5,585	41.0
2040	15.82	153.6	124.9	4,930	24.4
2045	16.20	134.9	106.4	4,360	12.5
		Monopoly: $\delta = .10$			
1975	13.09	62.8	43.8	10,956	321.7
1976	12.84	61.4	40.6	10,915	288.3
1977	12.65	60.9	38.6	10,877	242.8
1978	12.52	61.0	37.5	10,839	210.2
1979	12.42	61.6	37.0	10,803	185.2
1980	12.36	62.5	37.0	10,766	166.1
1985	12.35	70.2	41.7	10,569	114.9
1990	12.51	80.2	50.3	10,335	86.6
1995	12.71	91.7	61.3	10,052	66.0
2000	12.91	104.5	74.0	9.708	49.5
2005	13.12	118.7	88.5	9,295	36.5
2010	13.34	134.1	104.3	8,806	26.3
2015	13.59	150.2	120.8	8,235	18.4
2020	13.90	165.8	136.9	7,582	12.5
2025	14.28	178.6	150.0	6,857	8.0
2030	14.76	184.1	155.7	6,085	4.7
2035	15.32	176.4	148.0	5,324	2.4
2040	15.94	152.2	123.6	4,650	1.0
2045	16.46	118.8	90.1	4,133	0.4

	P	TD	D	R	Π_d[a]
		Competitive: $\delta = .05$			
1975	6.43	65.7	49.5	10,950	70.3
1976	6.50	66.3	50.6	10,900	71.3
1977	6.58	67.1	51.9	10,848	71.4
1978	6.65	68.2	53.4	10,795	71.7
1979	6.73	69.4	55.1	10,740	72.9
1980	6.81	70.9	56.8	10,683	73.9
1985	7.27	80.1	66.7	10,370	80.5
1990	7.80	91.9	78.2	10,002	86.5
1995	8.42	105.9	91.1	9,573	92.0
2000	9.15	122.1	105.8	9,075	96.7
2005	9.98	140.5	122.2	8,497	99.2
2010	10.92	160.7	140.2	7,832	99.8
2015	11.96	181.1	158.2	7,077	94.1
2020	13.07	196.7	171.1	6,243	81.1
2025	14.18	198.0	169.9	5,383	58.7
2030	15.20	176.6	146.1	4,596	32.3
2035	16.00	138.1	105.9	3,983	12.5
2040	16.51	101.2	67.8	3,572	3.2
2045	16.76	78.6	44.8	3,309	0.2
		(depletion occurs in 2066)			
		Competitive: $\delta = .10$			
1975	5.45	65.8	50.0	10,950	22.0
1976	5.50	66.5	51.6	10,898	21.3
1977	5.54	67.4	53.4	10,845	20.7
1978	5.59	68.6	55.3	10,790	20.5
1979	5.64	69.9	57.3	10,732	20.2
1980	5.70	71.4	59.4	10,673	20.2
1985	6.02	80.9	70.6	10,343	19.1
1990	6.42	92.8	83.0	9,953	17.8
1995	6.94	106.9	96.7	9,497	16.5
2000	7.62	123.4	112.0	8,969	15.4
2005	8.48	142.3	129.1	8,358	14.1
2010	9.58	163.8	148.1	7,656	12.6
2015	10.97	187.1	168.1	6,805	10.7
2020	12.66	205.7	182.8	5,836	8.1
2025	14.61	194.9	167.5	4,877	4.8
2030	16.75	129.5	97.3	4,205	1.9
2035	19.38	52.6	15.2	3,817	0.2
2040					
2045					
		(net demand becomes zero in 2038[b])			

[a] Millions of 1973 dollars.
[b] The optimal initial competitive price is somewhere between \$5.44 and \$5.45; at that price net demand would become zero at exactly the point that depletion occurred. The \$5.45 initial price is our closest approximation to the true optimal, and because it is slightly higher than the true optimal, net demand becomes zero before depletion occurs. This approximation has almost no effect, however, on the sum of discounted profits.

is, we must examine the sensitivity of our results to these parameter values. Bauxite is quite abundant; reserves for the competitive fringe can sustain production for nearly 300 years at current levels. Our competitive fringe supply function moves to the left only slowly as cumulative production increases—after a cumulative production of 1,700 million tons (e.g., 17 mmt/yr. for 100 years), supply would fall to 61% of its original value.

Equation (19) is the objective function for IBA. Initial reserves are 11,000 mmt (225 years of production at current levels), and initial production costs are $5 per ton. Initial conditions for the other variables are $TD_0 = 65.5$, $S_0 = 16.9$, and $CS_0 = 0$.

Optimal monopoly and competitive price trajectories are again calculated for discount rates of 0.05 and 0.10, but this time over an eighty-year horizon, since proven reserves are large. Optimal price trajectories are shown graphically in figure 3, and prices, total demand, IBA production, IBA reserves, and discounted profits are presented in table 3.

Observe that the optimal monopoly price has the typical characteristic of dropping for about five years and then rising slowly. The price fluctuates, however, over a small range, and at all times is within a few dollars of the "limit price" at which production of alumina from other ores becomes economical. The initial competitive price is again higher for the lower discount rate, but the difference in the initial percentage mark-ups above average cost for the two discount rates is much smaller, and the mark-ups themselves are much lower, than was the case for petroleum.[19] This is because the reserve base for bauxite is large (depletion in the competitive case takes at least 75 years) so that there is little incentive for competitive producers to withhold production in earlier years. In both the monopoly and competitive cases depletion plays only a small role in the determination of price during the first thirty years. Monopoly pricing is essentially "limit pricing" for a produced good; except for the effects of lag adjustments during the first five years, price can almost be chosen at the profit-maximizing "limit" each period, ignoring future periods.

The relative gains from cartelization are summarized in table 4. Again the relative gains are largest during the first five years, and are dependent on the discount rate. For either discount rate, however, the relative gains are larger than was the case for petroleum. Over the long term, cartelization of bauxite markets results in a 60% to 500% increase in the sum of discounted profits. This should be sufficient incentive for maintenance of the International Bauxite Association.

TABLE 4.—BAUXITE: RELATIVE PROFITS (ratio of *NPV* for monopolist to *NPV* for competitors)

	$NPV = \sum \Pi_d$ (1975 until exhaustion or 2050)
$\delta = .05$	$NPV_M / NPV_C = 7857/4835 = 1.63$
$\delta = .10$	$NPV_M / NPV_C = 3904/789 = 4.95$
	$NPV = \sum \Pi_d$ (first five years)
$\delta = .05$	$NPV_M / NPV_C = 1386/358 = 3.87$
$\delta = .10$	$NPV_M / NPV_C = 1248/105 = 11.89$

Because of the "limit pricing" characteristic of the monopoly solution, our results are not very sensitive to the elasticity assumptions that were used. Doubling the long-run elasticity of supply (which is the parameter about which we are most uncertain) results in optimal monopoly and competitive price trajectories that are always within 10% of the numbers reported in table 3.

It should be pointed out that IBA is currently selling bauxite at slightly above what we have calculated to be the optimal monopoly price. *In 1973 dollars*, bauxite is now selling in the range of $15 to $20. The actual limit price is probably higher than the $15.60 that we have calculated due to energy costs that have risen more rapidly than the general price index. We might thus expect the cartel to maintain its current price (in real terms) over the next twenty to thirty years.

V. Copper—The Gains to CIPEC[20]

Our model for copper draws heavily from the econometric model of the world copper market

[19] For bauxite $(P_0 - AC_0)/AC_0$ is 0.28 for $\delta = 0.05$ and 0.09 for $\delta = 0.10$ in the competitive case. For petroleum the corresponding numbers are 8.06 and 2.04.

[20] CIPEC countries include Chile, Peru, Zambia, and Zaire, and in 1974 accounted for 32% of non-Communist world copper production.

constructed by Fisher, Cootner, and Baily (1972), and in fact can be viewed as an aggregated version of their model. We also draw upon the work of Banks (1974), and obtain updated reserve and production data from the U.S. Bureau of Mines (1974b, 1975). The equations of the model are as follows:

$$TD_t = .405 - .78P_t + .90TD_{t-1} + .91(1.03)^t \tag{20}$$

$$SP_t = (-.190 + .8613P_t)\cdot(1.015)^{-CSP/4} + .88SP_{t-1} \tag{21}$$

$$CSP_t = CSP_{t-1} + SP_t \tag{22}$$

$$SS_t/K_t = .00940 + .005733P_t - .37(SS_{t-1}/K_{t-1}) \tag{23}$$

$$K_t = .98K_{t-1} + TD_t - SS_t \tag{24}$$

$$S_t = SP_t + SS_t \tag{25}$$

$$D_t = TD_t - S_t \tag{26}$$

$$R_t = R_{t-1} - D_t \tag{27}$$

$$\max W = \sum_{t=1}^{N} \frac{2204}{(1+\delta)^t}\left[P_t - \frac{67.5}{R_t}\right]D_t \tag{28}$$

where

TD_t = total demand for copper (mmt/yr.);
SP_t = primary supply from competitive fringe (mmt/yr.);
SS_t = secondary supply from scrap ("old" secondary) from competitive fringe (mmt/yr.);
CSP_t = cumulative primary supply of competitive fringe (mmt);
S_t = total supply from competitive fringe (mmt/yr.);
K_t = stock of copper in product form (mmt);
R_t = reserves of cartel (mmt);
P_t = price of copper ($ per pound) in constant 1974 dollars.

The demand equation (20) explains total demand for refined copper *net of "new" secondary production*.[21] The reason for excluding "new" secondary production will become clear shortly. Corresponding to average figures for 1974, total demand is 7.3 million metric tons at a price of $0.75 per pound.[22] At this price the short-run price elasticity is 0.16 and the long-run elasticity is 0.80. These elasticities correspond to a weighted average (weighted by consumption levels) of the regional demand elasticities estimated by Fisher, Cootner, and Baily.[23] The last term in equation (20) provides an autonomous rate of growth in demand of 3.75% per year, corresponding to a long-run income elasticity of 1.25 and a 3% real rate of growth in income.[24] The major substitute for copper is aluminum, but since we assume a fixed price for aluminum, an aluminum price need not be included explicitly in (20).

Primary supply from the competitive fringe is 3.8 mmt/yr. at a price of 75¢, with short-run and long-run elasticities of 0.20 and 1.6, respectively. Thus, although in the long-run supply is quite elastic, the adjustment time is considerable.[25] Depletion of competitive fringe reserves pushes the primary supply function to the left over time. After a cumulative production of 160 mmt (e.g., 4 mmt/yr. for 40 years), primary supply would fall (assuming a fixed price) to 55% of its original value.

Secondary supply includes only "old" secondary production, i.e., production from scrap, and thus depends on the stock of copper products available to be converted into scrap. The stock-adjustment effect will result in a price elasticity that is larger in the short run than in the long run. We follow Fisher,

[21] "New" secondary copper is produced from shavings and other wastage that result in the process of milling copper and producing finished copper products, while "old" secondary copper is produced from scrap; i.e., from recycled copper products.

[22] We assume that the same price prevails in all regions, and ignore differences between the U.S. producer price and the London Metal Exchange (LME) price. There have been periods during which the U.S. price was below the LME price, with rationing by U.S. producers. McNicol (1975) argues that rationing is profitable for partially integrated producers since it allows them to achieve the effects of price discrimination. The effect is therefore not that of undercutting a world market price to expand market share, and there would be no impact on the demand for cartel copper.

[23] Regional demand (and supply) equations for which price was statistically insignificant were not included in the average.

[24] The income elasticity is again an average of the Fisher, Cootner, Baily estimates.

[25] Again, this is based on the Fisher, Cootner, Baily estimates. More recent estimates by Banks (1974) indicate elasticities that are somewhat lower—around 1.0 in the long run.

Cootner, and Baily in using the primary price rather than the scrap price since the two prices are very highly correlated, and we apply their elasticity estimates for the United States (0.43 in the short run and 0.31 in the long run) to the rest of the world.[26] The stock of copper products available for scrap is simply last year's stock, minus losses of 2%, plus primary production this year; secondary production is not included since it adds to the stock of products but depletes the stock by the same amount.

As mentioned above, we exclude "new" secondary production from the model. The shavings and other "wastage" that provide the input to "new" secondary production come from the milling of copper sheet as well as the later transformation of sheet into a variety of copper products. Thus a good part of "new" secondary production is just a transformation of the output of primary production, and including it in total demand would involve some double counting. More important, although almost all secondary production occurs in the consuming countries, part of the input to "new" secondary production comes from CIPEC countries, so that including it would result in a misleading understatement of CIPEC's role in the world market.

The model is completed with equation (28), which specifies the maximization of the sum of discounted profits. The initial CIPEC proven reserve level is 135 mmt, and initial average cost is 50¢ per pound. The multiplication by 2,204 is to convert pounds to metric tons.

In calculating optimal price policies we take as initial conditions $TD_0 = 7.3$, $SP_0 = 3.8$, $SS_0 = 1.2$, $CSP_0 = 0$, and $K_0 = 120$. These numbers correspond to 1974 data. We use a forty-year horizon in the monopoly case, with discount rates of 0.05 and 0.10. Optimal monopoly and competitive prices are shown graphically in figure 4, and prices, total demand, competitive

[26] Outside of the United States data for secondary supply are not broken down into "new" and "old," and the determinants of "new" secondary supply are quite different. We obtain an estimate of the level of "old" secondary supply by applying the U.S. ratio of "old" secondary to total secondary outside the United States. Equation (17) represents a stock-adjustment model, so that the long-run elasticity is smaller than the short-run elasticity.

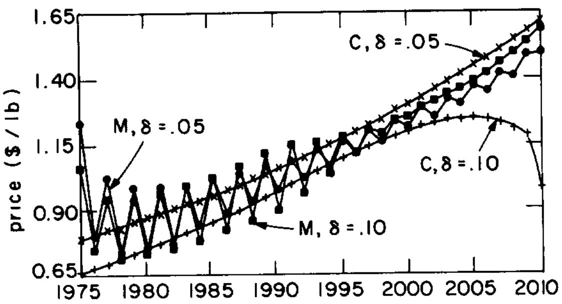
TABLE 5.—COPPER

	P	TD	SP	SS	D	R	Π_d[a]
				Monopoly: $\delta = .05$			
1975	1.23	6.92	4.22	1.43	1.28	133.8	2031
1976	0.78	6.97	4.19	1.14	1.64	132.2	964
1977	1.02	6.86	4.38	1.50	0.98	131.2	998
1978	0.73	6.99	4.29	1.20	1.50	129.7	657
1979	0.98	6.95	4.43	1.55	0.97	128.7	801
1980	0.75	7.13	4.36	1.28	1.50	127.2	570
1985	1.01	7.67	4.69	1.74	1.25	120.7	762
1990	0.97	8.59	5.03	1.78	1.78	112.8	701
1995	1.15	9.64	5.60	2.20	1.85	103.8	768
2000	1.20	10.99	6.20	2.42	2.38	93.0	735
2005	1.36	12.59	6.94	2.90	2.75	80.0	724
2010	1.49	14.47	7.77	3.36	3.34	64.4	590
				Monopoly: $\delta = .10$			
1975	1.06	7.06	4.06	1.41	1.59	133.4	1962
1976	0.75	7.11	4.03	1.17	1.91	131.5	957
1977	0.94	7.04	4.17	1.44	1.42	130.1	1112
1978	0.71	7.18	4.09	1.22	1.86	128.2	565
1979	0.94	7.16	4.22	1.52	1.42	126.8	876
1980	0.73	7.33	4.15	1.28	1.90	124.9	493
1985	1.02	7.85	4.52	1.79	1.54	116.4	576
1990	0.89	8.77	4.86	1.70	2.21	106.8	301
1995	1.17	9.74	5.52	2.27	1.94	96.9	300
2000	1.24	11.00	6.22	2.50	2.27	85.9	210
2005	1.38	12.48	7.06	2.93	2.48	74.0	147
2010	1.58	14.18	8.07	3.48	2.62	61.0	97
				Competitive: $\delta = .05$			
1975	0.79	7.27	3.83	1.23	2.21	132.8	1372
1976	0.80	7.26	3.88	1.28	2.10	130.7	1250
1977	0.82	7.26	3.93	1.32	2.02	128.7	1193
1978	0.83	7.28	3.98	1.36	1.94	126.7	1098
1979	0.85	7.32	4.05	1.41	1.87	124.9	1049
1980	0.87	7.37	4.12	1.45	1.81	123.1	1005
1985	0.95	7.83	4.55	1.67	1.61	114.7	788
1990	1.05	8.56	5.09	1.91	1.56	106.8	691
1995	1.16	9.52	5.74	2.19	1.59	99.0	632
2000	1.29	10.71	6.51	2.53	1.68	90.8	598
2005	1.44	12.14	7.39	2.94	1.81	82.0	569
2010	1.61	13.84	8.40	3.44	2.01	72.4	544
				(depletion occurs in 2031)			
				Competitive: $\delta = .10$			
1975	0.66	7.37	3.72	1.14	2.51	132.5	833
1976	0.68	7.45	3.67	1.22	2.56	129.9	823
1977	0.69	7.53	3.63	1.25	2.65	127.3	771
1978	0.71	7.63	3.62	1.31	2.70	124.6	750
1979	0.73	7.72	3.62	1.35	2.75	121.8	728
1980	0.75	7.83	3.64	1.40	2.79	119.0	698
1985	0.85	8.45	3.93	1.65	2.87	104.8	502
1990	0.96	9.21	4.45	1.93	2.83	90.5	320
1995	1.09	10.16	5.14	2.24	2.78	76.5	189
2000	1.23	11.37	5.88	2.57	2.91	63.4	65
2005	1.34	13.05	6.50	2.89	3.66	50.5	2
2010	1.25	15.08	6.51	2.94	5.63	24.8	0
				(depletion occurs in 2014[b])			

[a] Millions of dollars.

[b] The optimal initial competitive price is between 0.66 and 0.67. The 0.66 initial price is our closest approximation to the true optimal, and because it is slightly below the true optimal, price begins to decline and profits go negative before depletion occurs. The approximation has almost no effect, however, on the sum of discounted profits.

FIGURE 4.—OPTIMAL COPPER PRICE TRAJECTORIES
(M = monopoly, C = competition)



supply, CIPEC production, reserves, and discounted profits are presented in table 5.

We observe that the optimal monopoly price oscillates, and this occurs because of the stock adjustment effect in the secondary supply equation.[27] The envelope of the monopoly price, however, follows the same pattern as the monopoly petroleum and bauxite prices, dropping during the first several years (after taking advantage of lag adjustments in total demand and secondary supply), and then rising slowly as depletion of the resource base nears. Again the initial competitive price is higher for the lower discount rate. Resource exhaustion plays a more significant role in the determination of copper prices than was the case with bauxite; using U.S. Bureau of Mines figures for proven reserves, exhaustion in the competitive case occurs after 56 years at the 0.05 discount rate and after 39 years at the 0.10 discount rate.

TABLE 6.—COPPER: RELATIVE PROFITS
(Ratio of NPV for monopolist to NPV for competitors)

	$NPV = \sum \Pi_d$ (1975 until exhaustion or 2015)
$\delta = .05$	$NPV_M/NPV_C = 28988/26792 = 1.08$
$\delta = .10$	$NPV_M/NPV_C = 14772/11278 = 1.31$
	$NPV = \sum \Pi_d$ (first five years)
$\delta = .05$	$NPV_M/NPV_C = 5453/5962 = 0.91$
$\delta = .10$	$NPV_M/NPV_C = 5472/3905 = 1.40$

[27] The long-run price elasticity of secondary supply is smaller than the short-run elasticity. Price is set high in 1975 to take advantage of lag adjustments in total demand and primary supply, but this results in a large increase in secondary supply. If the 1975 price were maintained in 1976, secondary supply would fall because of the stock adjustment, but primary supply would rise further; dropping the price in 1976 results in a still larger drop in secondary supply.

As can be seen in table 6, the relative gains from cartelization are not large. Over the long term, cartelization of copper markets results in only an 8% to 30% increase in the sum of discounted profits. Furthermore, the increased profits require fluctuations in price (and in profits) that cartel members would probably wish to avoid, and that consuming countries could anticipate and counteract through stockpiling.[28] It would thus appear that there is little incentive for closely coordinated pricing and output policies on the part of the CIPEC cartel. This is consistent with recent history; so far CIPEC countries have not managed to significantly increase their copper revenues through cartelization.

VI. Concluding Remarks

We began this paper by asking whether the owners of exhaustible resources might accrue significant monopoly profits through cartelization. The answer would appear to be yes in the cases of petroleum and bauxite, and no in the case of copper. The reasons for this, however, have little to do with the fact that the resources are exhaustible, and more to do with market share and short-term lag adjustments. OPEC and IBA account for around two-thirds of non-Communist world petroleum and bauxite production, while CIPEC accounts for only one-third of copper production. Demand and competitive supply of petroleum and bauxite adjust only slowly to changes in price, allowing large short-term gains to a cartel, while secondary copper supply responds quickly to price changes.

Resource exhaustion, however, does have a significant effect on the pattern of pricing and output in both the monopoly and competitive cases, and tends to reduce the percentage increase in profits from cartelization when the discount rate is low. As competitive producers pay more attention to future depletion, they tend to restrict present output, raising prices closer to the monopoly level. The fact that pre-cartel oil, bauxite, and copper prices were close to marginal cost could mean that

[28] If the optimal monopoly price trajectory is smoothed out, the increase in profits from cartelization is reduced to only 3%–10%.

producers had large discount rates (which is reasonable at least for bauxite and copper), or that producers made output decisions either ignoring depletion or else assuming that large reserve additions would be discovered in the future.

In estimating the gains to producers from cartelization we have ignored a number of issues that could be important. Competitive fringe producers might form price expectations that would make their supply response different from that of equation (3) and would reduce cartel profits (alternative models of price expectations could easily be introduced into our framework by modifying equation (3)). Consuming countries might form a cartel of their own in an attempt to weaken the producer cartel; the behavior of price for such a bilateral monopoly could be analyzed in a dynamic game-theoretic framework, and would depend on the response assumptions made for each player. Producers—monopoly or competitive—might (and should) take uncertainty about reserves into account in making their price or output decisions, and this could alter cartel gains. The extent to which these issues would affect the conclusion of this paper will be determined from future research. Here we offer only a first approximation to the measurement of increased profits that could accrue to monopoly cartels in three exhaustible resource markets.

APPENDIX

The Competitive Price Trajectory with Rising Production Costs

We show here that equation (7) is a discrete-time approximation to the competitive price trajectory. Assume average cost is a function only of cumulative output x. Then, in continuous time, the competitive firm chooses output q_t to maximize

$$W=\int_0^T [q_t p_t e^{-\delta t} - C(x_t) q_t e^{-\delta t}]dt \tag{A.1}$$

with price p taken as given. This is subject to the constraints

$$\dot{x}_t = q_t \tag{A.2}$$

and

$$x_t \leqslant R_0. \tag{A.3}$$

Using the Maximum Principle, the Hamiltonian is

$$H=qpe^{-\delta t} - C(x)qe^{-\delta t}+\lambda q \tag{A.4}$$

and the Lagrangian is

$$L=H+\mu(R_0-x_t) \tag{A.5}$$

with $\mu \geqslant 0$ and $\mu(R_0-x)=0$, so that μ is zero until exhaustion occurs at time T. We have

$$\dot{\lambda}=C'(x)qe^{-\delta t}+\mu. \tag{A.6}$$

The optimal production trajectory q_t^* maximizes H, but since H is a linear function of q_t the problem is singular, so that H is maximized by $q_t=0$ or ∞ (or some upper limit q_M), depending on whether $pe^{-\delta t}-C(x)e^{-\delta t}+\lambda$ is negative or positive. λ is always negative (as we would expect, since it is the marginal discounted profit to go resulting from an additional unit of cumulative output) since $\dot{\lambda}$ is positive and λ approaches zero as exhaustion nears. Thus the decision to produce nothing or at the upper limit will depend on the price p. Then the market demand function will ensure that

$$pe^{-\delta t}-C(x)e^{-\delta t}+\lambda=0. \tag{A.7}$$

At the price that satisfies (A.7), producers will just be indifferent between producing nothing or everything, so that just enough will be produced to satisfy market demand. From (A.7) we have

$$\dot{\lambda}=C'(x)qe^{-\delta t}-\delta C(x)e^{-\delta t}+\delta pe^{-\delta t}-\dot{p}e^{-\delta t}. \tag{A.8}$$

Combining this with (A.6) and rearranging, we get

$$dp/dt=\delta[p-C(x)]. \tag{A.9}$$

This is the price solution to the continuous time problem. As a discrete-time approximation to this solution we use

$$p_t=(1+\delta)p_{t-1}-\delta C(x_{t-1}). \tag{A.10}$$

This equation, together with the boundary condition that exhaustion occurs at the same time that demand becomes zero, can be used to determine the competitive price path.

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